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Studying trilinear gauge couplings at linear collider energies

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Abstract. We investigate the sensitivity of the semileptonic processes $e^+e^- \to \ell^- \bar{\nu}_\ell \, q \, \bar{q}'$, $\ell = e$ or μ, on the non-standard trilinear gauge couplings, using the optimal observables method at Linear Collider energies. Our study is based on the four-fermion generator `ERATO`. Taking into account all possible correlations between the different trilinear gauge coupling parameters, we show that they can be measured with an accuracy of 10^{-3} to 10^{-4} for typical Linear Collider energies and luminosities.

The future e^+e^- linear colliders (LC), with energies ranging from a few hundreds of GeV up to a couple of TeV, provide particle physics with an enormous potential for studying Nature to the deepest level ever achieved [1]. Although at these energies direct searches for new particles and their interactions will eventually attract most of the physics interest, LC offer also the unique possibility to study to an extremely high accuracy, the properties of the existing particles, like those of the massive electroweak gauge bosons W and Z. Therefore, an important project at the LC energies will be the determination of the trilinear gauge couplings (TGC) [2,3], which are a characteristic manifestation of the underlying non-Abelian symmetry of elementary particle interactions [4] and at the same time an interesting probe of New Physics (NP).

In order to study the TGC we need a parameterization of the vector gauge boson interactions that goes beyond the Standard Model. The most general such parameterization is given by [5]:

$$\begin{aligned}
\mathcal{L}_{TGC} = \sum_{V=\gamma,Z} & -ie\, g_{VWW} \left(g_1^V (V_\mu W^{-\mu\nu} W_\nu^+ \right. \\
& \left. - V_\mu W^{+\mu\nu} W_\nu^-) + \kappa_V V_{\mu\nu} W^{+\mu} W^{-\nu} \right) \\
& - ie\, g_{VWW} \frac{\lambda_V}{m_W^2} V_{\mu\rho} W^{+\rho\nu} W_\nu^{-\mu} \\
& + e\, g_{VWW} g_5^V \varepsilon_{\mu\nu\rho\sigma} \left((\partial^\rho W^{-\mu}) W^{+\nu} \right. \\
& \left. - (\partial^\rho W^{+\nu}) W^{-\mu} \right) V^\sigma \\
& + e\, g_{VWW} \left[g_4^V W_\nu^+ W_\mu^- (\partial^\mu V^\nu + \partial^\nu V^\mu) \right. \\
& \left. + i\tilde{\kappa}_V W_\nu^+ W_\mu^- \mathcal{V}^{\mu\nu} + i \frac{\tilde{\lambda}_V}{m_W^2} W^{+\mu}{}_\nu W^-_{\rho\mu} \mathcal{V}^{\nu\rho} \right] \qquad (1)
\end{aligned}$$

where

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu, \quad W^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu,$$

and

$$\mathcal{V}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} V_{\rho\sigma}.$$

In (1) $W^\pm$ is the W-boson field, and the usual definitions $g_{\gamma WW} = 1$, $g_{ZWW} = \text{ctg}\, \theta_w$ are used. In the Standard Model we have $g_1^\gamma = g_1^Z = 1$, $\kappa_\gamma = \kappa_Z = 1$, while all the other parameters are vanishing at tree level. In searching for possible TGC, it is more convenient to express them in terms of their deviations from the Standard Model values. For this we define the deviation parameters [5,6]:

$$\begin{aligned}
&\delta_Z = (g_1^Z - 1) \text{ctg}\, \theta_w \qquad x_\gamma = \kappa_\gamma - 1 \\
&x_Z = (\kappa_Z - 1) \text{ctg}\, \theta_w - \delta_Z \quad ,
\end{aligned} \qquad (2)$$

while we throughout assume $g_1^\gamma = 1$, disregarding the possibility of an anomalous contribution to the electromagnetic form factor of $W^\pm$. We note that the NP contribution described by the interaction Lagrangian in (1), becomes linear when expressed in terms of the above deviation parameters and λ_γ, λ_Z, as well as the C- and P-violating couplings.

During the last years, considerable progress has been achieved concerning the understanding of the physics underlying the non-standard boson self-couplings. As showed in [7], the deviations from the Standard Model TGC couplings can be parameterized in a manifestly gauge-invariant way by using the effective Lagrangian approach and considering gauge-invariant operators involving higher-dimensional interactions among the gauge bosons and the Higgs field. These operators are scaled by an unknown parameter Λ_{NP} describing the characteristic scale of some high energy New Physics, generating at low energies the effective interaction $\mathcal{L}_{TGC}$ as a residual effect. In order to generate all kinds of TGC appearing in (1), we need operators with dimension up to twelve. On the other hand, restricting ourselves to $SU(2)_L \times U(1)_Y$-invariant operators with dimension six, which are the lowest order ones in a $1/\Lambda_{NP}$ expansion, we end up with the following list of operators capable to induce TGC NP couplings [8–11, 13,14]:

$$\begin{aligned}\mathcal{O}_{B\Phi} &= iB^{\mu\nu}(D_\mu\Phi)^\dagger(D_\nu\Phi)\\ \mathcal{O}_{W\Phi} &= i(D_\mu\Phi)^\dagger\, \boldsymbol{\tau}\cdot\boldsymbol{W}^{\mu\nu}(D_\nu\Phi)\\ \mathcal{O}_W &= \frac{1}{3!}(\boldsymbol{W}^{\mu}{}_{\rho}\times\boldsymbol{W}^{\rho}{}_{\nu})\cdot\boldsymbol{W}^{\nu}{}_{\mu}\end{aligned} \tag{3}$$

and[1]

$$\begin{aligned}\tilde{\mathcal{O}}_{BW} &= \Phi^\dagger\,\frac{\boldsymbol{\tau}}{2}\cdot\tilde{\boldsymbol{W}}^{\mu\nu}\Phi B_{\mu\nu}\\ \tilde{\mathcal{O}}_W &= \frac{1}{3!}(\boldsymbol{W}^{\mu}{}_{\rho}\times\boldsymbol{W}^{\rho}{}_{\nu})\cdot\tilde{\boldsymbol{W}}^{\nu}{}_{\mu}\ ,\end{aligned} \tag{4}$$

where

$$\tilde{B}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}B_{\rho\sigma}\quad ,\quad \tilde{\boldsymbol{W}}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma}\boldsymbol{W}_{\rho\sigma}\ . \tag{5}$$

In (3) and (4), τ_i describe the Pauli matrices,

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

is the $U(1)_Y$ gauge field strength,

$$\boldsymbol{W}_{\mu\nu} = \partial_\mu\boldsymbol{W}_\nu - \partial_\nu\boldsymbol{W}_\mu - g\boldsymbol{W}_\mu\times\boldsymbol{W}_\nu$$

is the field strength for the $SU(2)_L$ gauge field $\boldsymbol{W}_\mu$, and the Higgs doublet is written as

$$\Phi = \begin{pmatrix}\phi^+\\ \frac{1}{\sqrt{2}}(v+H+i\phi^0)\end{pmatrix}\quad ,$$

while D_μ is the covariant derivative

$$D_\mu = \partial_\mu + i\,g\frac{\boldsymbol{\tau}}{2}\cdot\boldsymbol{W}_\mu + i\,g'YB_\mu\quad ,$$

and Y is the hypercharge of the field on which D_μ acts. Finally $e = g\sin\theta_w = g'\cos\theta_w$.

In the list of (3, 4), we have included all `dim=6` purely bosonic operators contributing to the trilinear gauge interactions, except those which give also a tree level contribition to LEP1 observables, (and of course those which give no TGC at all). This constitutes part of a consistent general strategy for searching for any purely bosonic NP interaction. According to this strategy, the measurement of TGC provides the most efficient way to study the operators appearing in (3, 4), while the rest of `dim=6` operators can be most efficiently disentagled and constrained either by high precision measurements (LEP1), or by studying other production processes at LC [9,12,13] and high-energy hadronic colliders[2].

The New Physics contribution from the above operators is described by the effective Lagrangian

$$\begin{aligned}\mathcal{L}_{TGC} &= g'\frac{\alpha_{B\Phi}}{m_W^2}\mathcal{O}_{B\Phi} + g\frac{\alpha_{W\Phi}}{m_W^2}\mathcal{O}_{W\Phi} + g\frac{\alpha_W}{m_W^2}\mathcal{O}_W\\ &\quad + \frac{gg'}{2}\frac{\tilde{\alpha}_{BW}}{m_W^2}\tilde{\mathcal{O}}_{BW} + g\frac{\tilde{\alpha}_W}{m_W^2}\tilde{\mathcal{O}}_W\end{aligned} \tag{6}$$

where the relations between $\alpha_{W\Phi}$, $\alpha_{B\Phi}$, α_W, $\tilde{\alpha}_{BW}$, $\tilde{\alpha}_W$, and the deviation parameters of (2) are given by

$$\begin{aligned}&\delta_Z = \alpha_{W\Phi}/\left(\sin\theta_w\cos\theta_w\right)\qquad x_\gamma = \alpha_{B\Phi} + \alpha_{W\Phi}\\ &\lambda_\gamma = \alpha_W\qquad x_Z = -\tan\theta_w x_\gamma\qquad \lambda_Z = \lambda_\gamma\\ &\tilde{\kappa}_\gamma = \tilde{\alpha}_{BW}\qquad \tilde{\lambda}_\gamma = \tilde{\alpha}_W\\ &\tilde{\kappa}_Z = -\tan^2\theta_w\tilde{\kappa}_\gamma\qquad \tilde{\lambda}_Z = \tilde{\lambda}_\gamma .\end{aligned} \tag{7}$$

As it is seen from (7), the restriction to New Physics generated by `dim=6` gauge invariant operators, implies that there are only five independent non-standard triple gauge couplings, three of which are CP-conserving [15] and two CP-violating.

In order to study the effect of TGC, one usually considers the reaction $e^+e^- \to W^+W^-$, taking into account the subsequent decay of the two W's to a four-fermion final state [6]. Such final states can be classified in three categories, namely the 'leptonic' $\ell_1^-\bar{\nu}_{\ell_1}\ell_2^+\nu_{\ell_2}$, the 'semileptonic' $\ell^-\bar{\nu}_\ell\, q\bar{q}'$ and the 'hadronic' channel $q_1\bar{q}_1'\bar{q}_2q_2'$, (where q and q' refer to up- and down-type quarks respectively). Semileptonic seems to be the most favoured channel [5] for studying TGC, since it contains the maximum kinematical information; taking into account that charge-flavour identification in a four jet channel is rather inefficient and that the cross section for the leptonic mode is suppressed. Thus, in the present paper we study at LC energies, the TGC effect induced by the interaction (6) in the processes

$$e^+e^- \to \ell^-\bar{\nu}_\ell\; q\bar{q}' \tag{8}$$

where ℓ is an electron or a muon. The final state $\tau\bar{\nu}_\tau\; q\bar{q}'$ is not considered here, due to the special difficulties to identify τ's in this environment.

Quite often, the four-fermion final state processes, (8), are calculated by just taking into account contributions from the $e^+e^- \to W^+W^-$ subprocess, which is equivalent to a narrow width approximation $\Gamma_W \to 0$. In the classification of the four-fermion production diagrams of [16], these graphs are termed as the double-resonant graphs `CC03`. Such a narrow width approximation neglects contributions from single-resonant graphs, which become increasingly important at higher energies, (at least in some parts of the phase space) [16–19]. The situation is particularly severe for final states involving $e^\pm$, where graphs like the one presented in Fig. 1, which involves a t-channel photon exchange, dominate in certain parts of the phase-space at higher energies. Moreover, the graph of Fig. 1 receives contributions from the trilinear gauge boson interactions which are not included in the $e^+e^- \to W^+W^-$ calculation. In order to perform an analysis, as complete as possible, it is therefore mandatory to include in the

[1] The most complete list of CP violating `dim=6` purely bosonic operators is given in [13]. Concerning them we note that the TGC couplings generated by $\tilde{\mathcal{O}}_{BW}$, are identical to those induced by the operators $2\tilde{\mathcal{O}}_{B\Phi}/g$ or $2\tilde{\mathcal{O}}_{W\Phi}/g'$, defined as the CP violating analogs of $\mathcal{O}_{B\Phi}$ and $\mathcal{O}_{W\Phi}$ respectively

[2] This is particularly needed for the gluon involving operators; (see [14])

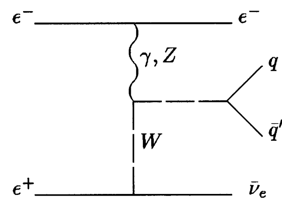

Fig. 1. Single-resonant graph where TGC are contributing

calculation of the processes (8) all tree-order diagrams, resonant as well as non-resonant ones.

Nowadays this is possible, since there exist widely available four-fermion codes, where the TGC effects are included beyond the narrow width approximation [5,17,18]. In the calculations presented in this paper we have used for this purpose, the `ERATO` Monte-Carlo event generator described in [16,17,20]. The basic ingredients of this generator are the following:

1. Exact tree-order helicity amplitudes for the processes $e^+e^- \to \ell^- \bar{\nu}_\ell \, q\bar{q}'$, including all trilinear gauge interactions described by (1) [17,21].
2. Phase-space generation algorithm based on a multichannel Monte Carlo approach, including weight optimization [22].
3. Treatment of the unstable particles contribution consistent with gauge-invariance and high-energy unitarity [17,23,24].
4. Initial state radiation (ISR), based on the structure function approach [25], including soft-photon exponentiation as well as hard collinear photon emission in the leading logarithmic (LL) approximation up to order $O(\alpha^2)$.
5. Coulomb correction[3] to the double resonant (`CC03`) graphs.
6. Beamstrahlung effects have also been included via the '$\kappa\acute{\iota}\varrho\kappa\eta$' algorithm [26].

Apart from the beamstrahlung effects just mentioned, the treatment of the higher order corrections in the present study is the same as in the LEP2 case.

In order to avoid matrix element singularities and to be as close as possible to the experimental situation, we have applied the cuts

$$175^o \geq (\theta_\ell \ , \ \theta_{jet}) \geq 5^o, \quad E_\ell \geq 10 \text{ GeV} \ , \\ E_{jet} \geq 10 \text{ GeV} \ \text{ and } \ m_{q,\bar{q}'} \geq 15 \text{ GeV} \ . \tag{9}$$

Finally, we use the Standard Model input parameters

$$M_W = 80.23 \text{ GeV}, \qquad \Gamma_W = 2.033 \text{ GeV}, \\ M_Z = 91.188 \text{ GeV}, \qquad \Gamma_Z = 2.4974 \text{ GeV}, \\ \sin^2\theta_w = 0.23103 \ \text{ and } \alpha(M_Z) = 1/128.07 \ , \tag{10}$$

while in the ISR structure function $\alpha = 1/137.036$ is of course used. For the analysis of the beamstrahlung effects we have used the `TESLA` design.

In order to determine the sensitivity of a given reaction on the TGC parameters one has to maximize the likelihood function [28], whose logarithm is given by

$$\ln \mathcal{L}_{ML} = \sum_i^N \ln p(\Omega_i, \mathbf{a}) \ , \tag{11}$$

where the sum is running over the event sample under investigation. Ω_i represents the collection of the independent kinematical variables describing the i-th event, the vector $\mathbf{a}$ is defined in the coupling space as $\mathbf{a} = (\alpha_{W\Phi}, \alpha_{B\Phi}, \alpha_W, \tilde{\alpha}_{BW}, \tilde{\alpha}_W)$, and

$$p(\Omega_i, \mathbf{a}) = \frac{1}{\sigma}\frac{d\sigma}{d\Omega}\Big|_{|\Omega=\Omega_i} \ , \tag{12}$$

$$\sigma = \int_V \frac{d\sigma}{d\Omega} d\Omega \ , \tag{13}$$

is the probability to find an event at the phase-space point Ω_i. Since the interaction Lagrangian is linear with respect to the TGC parameters, one can write the differential cross section in the form

$$\frac{d\sigma}{d\Omega} = c_0(\Omega) + \sum_i a_i c_{1,i}(\Omega) + \sum_{i,j} a_i a_j c_{2,ij}(\Omega) \tag{14}$$

and similarly the total cross section as

$$\sigma = \hat{c}_0 + \sum_i a_i \hat{c}_{1,i} + \sum_{i,j} a_i a_j \hat{c}_{2,ij} \ , \tag{15}$$

where hatted c's are integrals of unhatted ones over the phase space.

The sensitivity on the TGC parameters is determined by the so-called information matrix [29] given by the second derivative of the likelihood function,

$$I_{ij} \equiv E\left[\left(\frac{\partial}{\partial a_i}\ln \mathcal{L}_{ML}\right)\left(\frac{\partial}{\partial a_j}\ln \mathcal{L}_{ML}\right)\right] \\ = -E\left[\frac{\partial}{\partial a_i}\frac{\partial}{\partial a_j}\ln \mathcal{L}_{ML}\right] \tag{16}$$

where

$$E[A] = \int \prod_{i=1}^N \{d\Omega_i \ p_0(\Omega_i)\} A(\Omega_1, ..., \Omega_N) \tag{17}$$

represents the mean value of a function $A(\Omega_1, ..., \Omega_N)$. If we assume that the maximum of the likelihood function is located at $\mathbf{a} = 0$, which reflects the physical expectation that the 'data' will be consistent with the Standard Model, the information matrix is given to the lowest order, by $I_{ij} = N\mathcal{B}_{ij}$, with

$$\mathcal{B}_{ij} \equiv \left\langle \frac{c_{1,i}}{c_0}\frac{c_{1,j}}{c_0}\right\rangle_0 - \left\langle \frac{c_{1,i}}{c_0}\right\rangle_0 \left\langle \frac{c_{1,j}}{c_0}\right\rangle_0 \tag{18}$$

and

$$\langle A \rangle_0 = \int d\Omega \ p_0(\Omega) \ A(\Omega) \ , \tag{19}$$

[3] For a detailed description see [25]

$$p_0(\Omega) = \frac{1}{\sigma}\frac{d\sigma}{d\Omega}\bigg|_{\mathbf{a}=0} \quad . \tag{20}$$

In the optimal observables approach [30], equivalent results are obtained by defining the phase-space functions

$$\mathcal{O}_i = \frac{c_{1,i}(\Omega)}{c_0(\Omega)} \tag{21}$$

called optimal observables, whose mean values and covariance matrix determine the sensitivity on the TGC parameters. More specifically one writes

$$\bar{a}_i = \sum_j \mathcal{B}^{-1}_{ij} \left(\langle \mathcal{O}_j \rangle - \langle \mathcal{O}_j \rangle_0\right)$$

as an unbiased estimator of the components of $\mathbf{a}$, while the corresponding covariance matrix is given by

$$V(\bar{\mathbf{a}}) = \frac{1}{N}\ \mathcal{B}^{-1} \cdot V(\mathcal{O}) \cdot \mathcal{B}^{-1}$$

with $V(\mathcal{O})$ defined by

$$V(\mathcal{O})_{ij} = \langle \mathcal{O}_i \mathcal{O}_j \rangle - \langle \mathcal{O}_i \rangle \langle \mathcal{O}_j \rangle \ ,$$

and

$$\langle A \rangle = \int d\Omega\ p(\Omega)\ A(\Omega) \ \ .$$

Under the assumption that the 'data' are accounted for by the Standard Model, we have that $V(\mathcal{O}) = \mathcal{B}$, which shows that to the lowest order, the likelihood approach and the optimal observables are indeed equivalent.

Up to now we have considered the so called Maximum Likelihood method. One can improve the analysis by considering also the so called Extended Maximum Likelihood (EML). In this case we take into account the variation of the total number of expected events as a function of the unknown parameters and define

$$\mathcal{L}_{EML} = p_N \prod_i^N p(\Omega_i, \mathbf{a}) \ \ , \tag{22}$$

where

$$p_N = \frac{\langle N \rangle^N}{N!} e^{-\langle N \rangle}$$

and $\langle N \rangle$ is the number of expected events. The analysis proceeds as before and the correlation matrix is expressed as

$$\mathcal{B}_{i,j} = \left\langle \frac{c_{1,i}}{c_0} \frac{c_{1,j}}{c_0} \right\rangle_0 \tag{23}$$

while the information matrix becomes $I_{ij} = \langle N \rangle \mathcal{B}_{i,j}$. The corresponding optimal observables in the EML approach are

$$\mathcal{O}_i = N\ \frac{c_{1,i}(\Omega)}{c_0(\Omega)} \ \ . \tag{24}$$

In the sequel we perform one- as well as five-dimensional investigations. One-dimensional investigations assume that all but one of the a_i's are non-vanishing, and lead to parameter errors (1sd) given by

$$\delta a_i = \frac{1}{\sqrt{N b_{ii}}} \ \ . \tag{25}$$

The multidimensional case where all five NP couplings are considered simultaneously, is also treated. Diagonalizing then the symmetric correlation matrix $\mathcal{B}$, one first determines its eigenvalues λ_i and eigenvectors $\mathbf{e}_i$, normalized so that $\mathbf{e}_i \cdot \mathbf{e}_j = \delta_{ij}$. For each $\mathbf{e}_i$, the parameter

$$a_i^D \equiv \mathbf{e}_i \cdot \mathbf{a} \tag{26}$$

is then defined, for which the (1sd) error is given by

$$\delta a_i^D = \frac{1}{\sqrt{N \lambda_i}} \ . \tag{27}$$

In all calculations presented in this paper, N is taken to be the predicted Standard Model number of events defined by $N = 4\ L\ \sigma_{SM}$, where σ_{SM} is the corresponding total cross section, L is the integrated luminosity and the factor 4 takes into account the four equivalent channels described by the same matrix elements; i.e. $e^+e^- \to \ell^- \bar{\nu}_\ell\ u\bar{d}$, $\ell^- \bar{\nu}_\ell\ c\bar{s}$, $\ell^+ \nu_\ell\ d\bar{u}$ and $\ell^+ \nu_\ell\ s\bar{c}$.

At this point we address the question, how accurately the optimal observables approximation describes the ML (or EML) function. To this end we calculate the $\ln \mathcal{L}_{ML}$ by replacing the sum appearing in (11) by an integral over the expected probability distribution which is assumed to be the one predicted by the Standard Model

$$\ln \mathcal{L}_{ML} = N \int \prod_{i=1}^N \{d\Omega_i\ p_0(\Omega_i)\} \ln p(\Omega_i, \mathbf{a}) \ \ . \tag{28}$$

It is clear that optimal observables and ML methods become identical in the limit $N \to \infty$, since then only the leading term in the expansion of the likelihood function survives; and this is exactly the term proportional to the information matrix. On the other hand for relatively low statistics, the nonlinearity of the likelihood function becomes important and the optimal observables approximation breaks down. These features are shown in Fig. 2, where the one, and two standard deviation limits on ($\alpha_{B\Phi}$, $\alpha_{W\Phi}$) are considered, for the muon channel and $\sqrt{s} = 800$ GeV. In the upper part of the figure the value of the integrated luminosity is taken to be $L = 50$ fb^{-1}, which is the expected nominal value, whereas in the lower part a much lower luminosity, $L = 5$ fb^{-1}, has been used.

We have checked that for all nominal LC energies and luminosities, the optimal observables approximation gives identical results to those of the conventional likelihood approach. Furthermore the fact that the expected sensitivities on the TGC parameters are predominantly determined by the linear terms in the expansion of the differential cross section, (14), shows the self-consistency of our original assumption that a parameterization of TGC in terms of `dim=6` operators should be adequate.

On the other hand, from the point of view of a weighted Monte-Carlo approach, which is frequently used in the

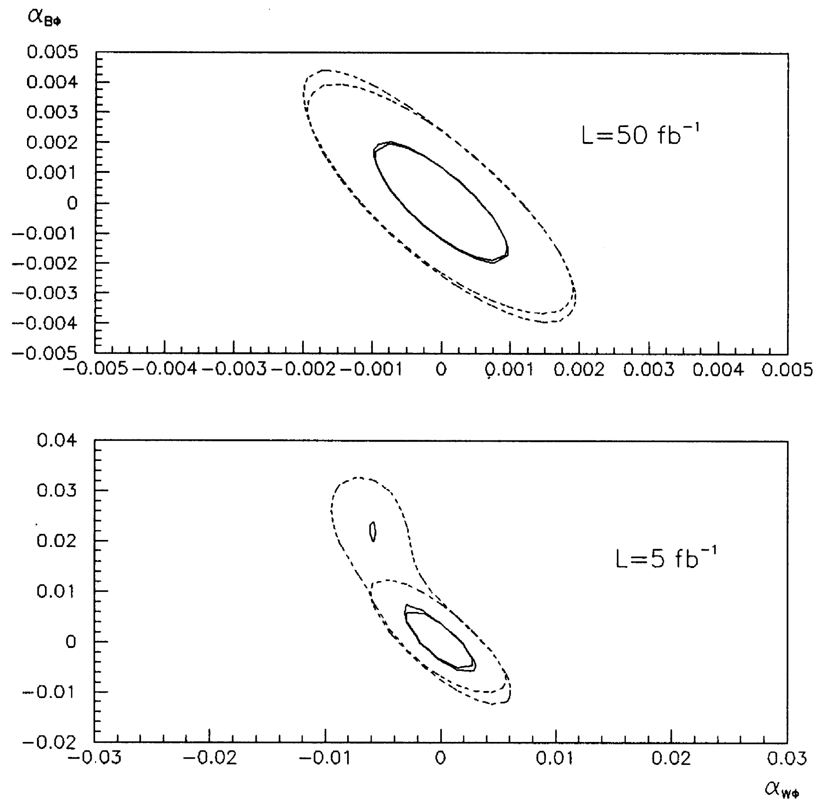

Fig. 2. The one (*solid*) and two (*dashed*) standard deviations limits on $\alpha_{B\Phi}$-$\alpha_{W\Phi}$, using the optimal observables and the EML methods for $\sqrt{s} = 800$ GeV and unpolarized beams. In the upper part optimal observables and EML are hardly distinguishable, whereas in the lower part EML exhibits a secondary minimum

phenomenological analyses, the optimal observables method offers a very efficient fast and economic way to estimate not only the sensitivity of a given process on a single TGC parameter (or any kind of 'deviation' parameter), but also their full covariance matrix, which is of great importance for multiparameter analyses.

Finally we would like to mention that the general experimental problem of how to overcome the ISR as well as the detector resolution induced difficulties in the reconstruction of the event kinematics, is not addressed here. We only note that this problem exists also for the current LEP2 experiments and that detailed experimental simulations at linear collider energies can be found in the proceedings of the DESY-ECFA Workshop on Linear Colliders [27]. Moreover our experience from LEP2 studies [5] shows that, despite the abovementioned problem, a very good estimate of the sensitivity on the TGC can be obtained by an analysis of the kind used in our present study.

In Table 1 we present the results for the correlation matrix $\mathcal{B}_{ij}$ involving all CP-conserving and CP-violating couplings, at 500 GeV center of mass energy. The total cross sections are also presented. As is evident from this table, the correlations between the different a_i's are not negligible in general, which suggests that an analysis taking them into account, is indispensable.

Another very interesting result, is that electron and muon channels exhibit a complementary behaviour: electron channel gives the highest production rate, which means a better statistics, whereas the muon-channel exhibits a higher sensitivity on TGC.

In Table 2 we show the eigenvalues of the correlation matrix, as well as the corresponding eigenvectors. These eigenvectors define directions in the five-parameter space, which are uncorrelated, so that parameter errors can be safely extracted. This table shows that for the unpolarized beams case, the dominant eigenvalues correspond to directions in the five-parameter space related predominantly to α_W, $\alpha_{W\Phi}$, and $\tilde{\alpha}_W$, whereas the lowest ones are related to $\alpha_{B\Phi}$ and $\tilde{\alpha}_{BW}$. The picture becomes almost opposite in the case that the electron (positron) beams are right-(left)-polarized.

As far as the polarization is concerned, we see that passing from unpolarized to right-polarized electron-beam, results to a much higher sensitivity for $\alpha_{B\Phi}$ and $\tilde{\alpha}_{BW}$ couplings. These phenomena are much more pronounced for the muon-channel. This effect, which has been also observed in on shell studies of $e^-e^+ \to W^-W^+$ [32], reflects the fact that different TGC parameters contribute to different helicity amplitudes, especially in the high-energy regime. In Fig. 3 we show how the information from both polarizations can, in principle, be used to disentangle different TGC parameters, based on the fact that the corresponding covariance matrices are very different.

Finally in Table 3 one-standard-deviation errors are presented by combining electrons and muons as

$$\mathcal{B}_{ij} = \mathcal{B}_{ij}^{(e)} \frac{\sigma^{(e)}}{\sigma^{(e)} + \sigma^{(\mu)}} + \mathcal{B}_{ij}^{(\mu)} \frac{\sigma^{(\mu)}}{\sigma^{(e)} + \sigma^{(\mu)}} \quad ,$$

$$N = 4L(\sigma^{(e)} + \sigma^{(\mu)}) \quad .$$

In our studies, L is taken to be 20 fb^{-1} at 500 GeV, 10 fb^{-1} at 360 GeV and 50 fb^{-1} at 800 GeV. For the results

Table 1. The correlation matrix for e and μ channels at 500 GeV for the TGC parameters $\alpha_{W\Phi}, \alpha_{B\Phi}, \alpha_W, \tilde{\alpha}_{BW}, \tilde{\alpha}_W$. Also shown are the cross sections as well as their Monte Carlo errors in femptobarns. Here $\mathcal{P}_{RL}(\mathcal{P}_{LR})$ corresponds to $e_R^- e_L^+$ $(e_L^- e_R^+)$ initial state polarization

	$\mathcal{P}_{LR}+\mathcal{P}_{RL}$					$\mathcal{P}_{RL}$				
e	14.53	4.87	3.26	-0.0032	-0.27	9.66	-10.20	0.33	0.59	0.027
	4.87	3.67	0.33	0.016	-0.072	-10.20	70.33	-0.11	-1.80	-0.096
	3.26	0.33	20.75	-0.011	0.38	0.33	-0.11	3.50	0.012	-0.016
	-0.0032	0.016	-0.01	0.28	-0.78	0.59	-1.80	0.012	6.66	0.22
	-0.27	-0.072	0.38	-0.78	21.53	0.027	-0.096	-0.016	0.22	3.46
			456(5)					207(5)		
μ	23.33	7.33	5.66	0.013	-0.060	258.42	-513.87	12.94	-1.35	-0.12
	7.33	5.25	0.71	0.033	-0.054	-513.87	2478.22	-9.50	12.36	0.28
	5.66	0.71	33.00	-0.032	0.29	12.94	-9.50	2.71	0.27	-0.0013
	0.013	0.033	-0.032	0.28	-1.33	-1.35	12.36	0.27	178.16	12.11
	-0.060	-0.054	0.29	-1.33	33.85	-0.12	0.28	-0.0013	12.11	2.67
			267(2)					6.01(4)		

Table 2. The eigenvalues (2nd and 8th columns) and the corresponding eigenvectors of the correlation matrices given in Table 1

	e						μ					
$\mathcal{P}_{LR}$	22.49	-0.425	-0.124	-0.868	0.008	-0.220	36.10	-0.460	-0.129	-0.872	0.004	-0.099
+	21.54	0.149	0.0477	0.167	0.035	-0.972	33.90	0.003	0.002	-0.007	0.039	-0.999
$\mathcal{P}_{RL}$	14.71	0.813	0.344	-0.463	-0.002	0.061	22.88	0.816	0.320	-0.479	0.0005	0.018
	1.77	-0.366	0.929	0.046	0.011	-0.002	2.60	-0.343	0.938	0.041	0.010	0.001
	0.25	0.004	-0.010	-0.0006	0.999	0.036	0.22	0.002	-0.010	0.0007	0.999	0.039
$\mathcal{P}_{RL}$	72.06	0.161	-0.986	0.002	0.028	0.001	2591.53	0.215	-0.976	0.004	-0.005	-0.0001
	8.07	-0.963	-0.163	-0.067	-0.198	-0.011	178.98	-0.040	-0.003	-0.004	-0.996	-0.068
	6.56	-0.199	-0.0043	-0.018	0.977	0.068	145.96	0.973	0.214	0.073	-0.040	-0.003
	3.48	0.062	0.007	-0.894	-0.034	0.440	1.87	0.068	0.010	-0.935	-0.022	0.346
	3.44	-0.028	-0.003	0.440	-0.059	0.895	1.83	-0.024	-0.003	0.346	-0.064	0.935

concerning polarized beam scattering, we used

$$L_{\text{polarized}} = \frac{1}{4}\, L_{\text{unpolarized}}\,.$$

In order to study the effect of the correlations among the TGC parameters we distinguish two cases:

- The $1d$-case is based on the very strong and often made assumption that only one of the TGC parameters ($\alpha_{W\Phi}$, $\alpha_{B\Phi}$, α_W, $\tilde{\alpha}_{BW}$, $\tilde{\alpha}_W$) is non vanishing. A very contrived form of NP at high energy scale is needed, in order to create such a situation where only one of the operators appearing in (3,4) is generated at low energies [31]. This case corresponds to the 'one-dimensional log-likelihood fit'.
- In the $5d$-case, on the contrary, the errors are calculated according to (27), where the full correlation matrix is taken into account and no *a priori* assumption on the size of the parameters has been made. Although in this case the presented errors correspond to directions in the five-parameter space defined by (26), which are not generally identical to the ones defined by the original parameters, we kept the same notation, since the former are rather close to the latter: for instance $\alpha^D_{W\Phi}$ is mainly composed by $\alpha_{W\Phi}$ and so on for the other TGC parameters [33].

As it is seen from Table 3, moving from the $1d$-case to the $5d$-case, the change on the one-standard-deviation errors reach the level of 40%. Moreover the less sensitive the TGC parameter is, the more the correlations affect its error. It should be mentioned however that the correlations among the different TGC parameters do not dramatically change the order-of-magnitude estimate of their sensitivity based on single-parameter considerations.

Concerning the CP violating interactions, we should note that in [34], bounds on the CP-violating couplings $\tilde{\kappa}_\gamma$ and $\tilde{\lambda}_\gamma$ have been derived, on the basis of their contri-

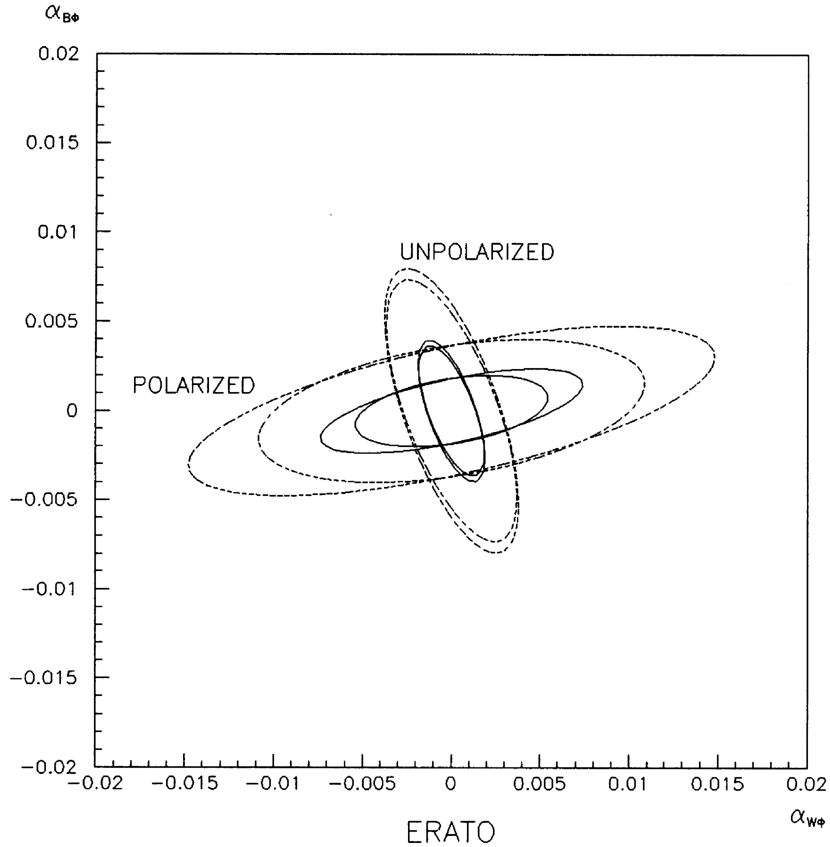

Fig. 3. The one (*solid*) and two (*dashed*) standard deviations limits on $\alpha_{B\Phi}$-$\alpha_{W\Phi}$ for unpolarized and polarized ($e^-_R e^+_L$) beams. The inner lines are from electron channel whereas the outer ones are from muon channel

Table 3. One standard deviation errors on TGC parameters. At 500 GeV we show also the effect of the correlations between different five TGC parameters in the $5d$ case for unpolarized beams, whereas in parentheses we show the corresponding errors from a right-handed polarized (left-handed) electron (positron) beam, as explained in the text

$\sqrt{s}$ (GeV)	360	500		800
	1d	1d	5d	1d
$\alpha_{W\Phi}$	0.0018	0.00098 (0.0037)	0.00098 (0.0045)	0.00042
$\alpha_{B\Phi}$	0.0039	0.0020 (0.0013)	0.0028 (0.0012)	0.00083
α_W	0.0016	0.00082 (0.0082)	0.00081 (0.0082)	0.00031
$\tilde{\alpha}_{BW}$	0.011	0.0078 (0.0045)	0.0084 (0.0044)	0.0048
$\tilde{\alpha}_W$	0.0016	0.00081 (0.0082)	0.00079 (0.0083)	0.00031

bution to the electric dipole moment (EDM) of the meutron. Besides the fact that these bounds depend on several details, the most they imply is a strong linear relation between $\tilde{\kappa}_\gamma$ and $\tilde{\lambda}_\gamma$. Direct measurements of these couplings, as well as of their Z-boson counterparts, will therefore be useful because they will provide detail information on the whole CP-violating TGC parameter-space.

In studying the sensitivity on the TGC one usually neglects possible correlations with other electroweak parameters like for instance the non-standard contributions to $Vf\bar{f}$ vertices, where V stands for Z or W. This is rather well justified because the latter are usually much more constrained than the former as it is indeed the case at LEP2, where TGC determination is expected to reach the level of 0.01 to 0.1, compared with the constraints on the $Vf\bar{f}$ couplings coming mainly from LEP1 analysis, which are at the level of 10^{-3} [35]. On the other hand, as it is also suggested by our analysis, at LC energies the TGC can be tested to a precision much higher than that of LEP2, and therefore it becomes interesting to study the correlations among the TGC and the other electroweak couplings [36, 37].

Finally we would like to mention that a detailed comparison of our study with those presented in [6,32] based on the on-shell production $e^-e^+ \to W^-W^+$ is not possible, due mainly to the fact that we are using different analysis methods. Nevertheless both approaches agree in the order-of-magnitude estimate of the expected sensitivity on the TGC.

We conclude by summarizing the results of our study:

1. We have presented a five-parameter description of the non-standard trilinear gauge couplings, which includes all gauge-invariant contributions to the lowest order. We then analysed their contribution to the semileptonic reactions $e^+e^- \to \ell^-\bar{\nu}_\ell\, q\bar{q}'$ for $\ell = e$ and $\ell = \mu$ at LC energies, and showed that a measurement of all five parameters is possible, with a sensitivity covering a rather wide range starting from 1.1×10^{-2} (1sd)

at 360 GeV for $\tilde{\alpha}_{BW}$, (worst case), and going down to 3×10^{-4} (1sd) at 800 GeV for α_W and $\tilde{\alpha}_W$, (best case).
2. The electron channel, due to the onset of the single-W production mode, gives the dominant contribution to the total cross section at LC energies, whereas the muon channel exhibits a higher sensitivity on the TGC parameters. Therefore their overall contribution to the error on the TGC determination become equally important.
3. Polarization effects are important in order to disentangle different TGC contributions, leading to a substantial improvement of the sensitivity on the TGC parameters, especially for $\alpha_{B\Phi}$ and $\tilde{\alpha}_{BW}$.

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